

## Section 6.5: The Inverse Trig. Functions

1- **The inverse Sine function:** denoted by  $\sin^{-1}$  or arcsin is defined by

$$y = \sin^{-1} x \quad \text{if and only if} \quad x = \sin y$$

$$\text{for} \quad -1 \leq x \leq 1 \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

### Properties:

$$\text{a- } \sin(\sin^{-1} x) = x \quad \text{if} \quad -1 \leq x \leq 1$$

$$\text{b- } \sin^{-1}(\sin x) = x \quad \text{if} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

**Examples:** Find the exact value of the followings;

1)  $\sin^{-1}(\frac{1}{2})$  , 2)  $\sin^{-1}(-\frac{\sqrt{3}}{2})$  , 3)  $\sin^{-1}(-1)$  , 4)  $\sin(\sin^{-1} \frac{1}{\sqrt{2}})$  , 5)  $\sin(\sin^{-1} \pi)$  ,

6)  $\sin^{-1}(\sin \frac{\pi}{4})$  , 7)  $\sin^{-1}(\sin \frac{2\pi}{3})$  , 8)  $\sin^{-1}(\tan \frac{3\pi}{4})$  , 9)  $\sin^{-1}(\sin(\frac{11\pi}{10}))$  ,

10)  $\tan(\sin^{-1}(-\frac{2}{3}))$  , 11)  $\cot[\sin^{-1}(1) - \sin^{-1}(-\frac{1}{\sqrt{2}})]$

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2- **The inverse Cosine function:** denoted by  $\cos^{-1}$  or arccos is defined by

$$y = \cos^{-1} x \quad \text{if and only if} \quad x = \cos y$$

$$\text{for} \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi$$

### Properties:

$$\text{a- } \cos(\cos^{-1} x) = x \quad \text{if} \quad -1 \leq x \leq 1$$

$$\text{b- } \cos^{-1}(\cos x) = x \quad \text{if} \quad 0 \leq x \leq \pi$$

**Examples:** Find the exact value of the followings;

1)  $\cos^{-1}(-\frac{\sqrt{3}}{2})$  , 2)  $\cos^{-1}(-\frac{1}{\sqrt{2}})$  , 3)  $\cos^{-1}(-1)$  , 4)  $\cos(\cos^{-1} \frac{1}{5})$  , 5)  $\cos(\cos^{-1}(-\sqrt{2}))$  ,

6)  $\cos^{-1}(\cos \frac{7\pi}{6})$  , 7)  $\cos(\cos^{-1}(-\frac{1}{a}))$  ,  $a > 1$  , 8)  $\cos^{-1}(\sin(-\frac{\pi}{6}))$  , 9)  $\sin(\cos^{-1}(-\frac{1}{3}))$  ,

10)  $\cos^{-1}(\cos(\frac{11\pi}{9}))$ , 11)  $\cos^{-1}0 - \cos^{-1}(-\frac{1}{2})$ , 12)  $\csc[\sin^{-1}(-1) - \cos^{-1}(0)]$

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3- **The inverse tangent function:** denoted by  $\tan^{-1}$  or  $\arctan$  is defined by

$$y = \tan^{-1} x \quad \text{if and only if} \quad x = \tan y$$

$$\text{for} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

**Properties:**

a-  $\tan(\tan^{-1} x) = x \quad \text{if} \quad -\infty < x < \infty$

b-  $\tan^{-1}(\tan x) = x \quad \text{if} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

**Examples:** Find the exact value of the followings;

1)  $\tan^{-1}(-\frac{1}{\sqrt{3}})$ , 2)  $\tan^{-1}(1)$ , 3)  $\tan^{-1}(\tan 0.63)$ , 4)  $\tan(\tan^{-1} 43)$ , 5)  $\sin(\tan^{-1}(-1))$ ,

6)  $\sec(\tan^{-1}(-\frac{2}{3}))$ , 7)  $\tan(\sin^{-1}(-\frac{a}{b}))$ ,  $b > a$ , 8)  $\sin(\cos^{-1}(-\frac{1}{a}))$ ,  $a > 1$

9)  $\tan^{-1}(\tan \frac{13\pi}{8})$ , 10)  $\sec[\tan^{-1}(-\frac{1}{\sqrt{3}}) - \cos^{-1}(-1)]$ , 11)  $\cos^{-1}(-\frac{1}{2}) - \frac{1}{2} \sin^{-1}(-\frac{1}{\sqrt{2}})$

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**Examples:** Solve the following inverse equation

1)  $\tan^{-1}(2x + \sqrt{3}) = -\frac{\pi}{3}$ , 2)  $\cos^{-1}(\frac{3}{2} - \frac{x}{3}) = \frac{2\pi}{3}$ , 3)  $\sin^{-1}(\sqrt{2} - \frac{x}{2}) = -\frac{\pi}{4}$

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**Examples:** True or False

1) Since  $\tan(-\frac{\pi}{4}) = -1$  then  $\tan^{-1}(-1) = -\frac{\pi}{4}$

2) Since  $\cos(\frac{3\pi}{2}) = 0$  then  $\cos^{-1}(0) = \frac{3\pi}{2}$

3)  $\tan^{-1}(\tan x) = x$  given  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

4) Since  $\sin(\pi) = 0$  then  $\sin^{-1}(0) = \pi$

5)  $\cos^{-1}\left(-\frac{2}{\sqrt{2}}\right)$  is in quadrant II

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**Examples:** Evaluate the followings

1)  $\csc\left[\frac{7\pi}{2} - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

2)  $\tan\left[-\frac{11\pi}{6} - \cos^{-1}\left(-\frac{1}{2}\right)\right]$

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