## Section 7.2: The Law of Sines

If none of the angles of a triangle is right angle. The triangle is called
Oblique.


## To Solve Oblique Triangle:

## Case I :



SAA or ASA

Case II:


SSA ( angle opposite to one of the sides)

Use law of Sine

## Law Of Sines:


$\frac{\sin \alpha}{a}=\frac{\sin \beta}{b}=\frac{\sin \gamma}{c}$
Ex: In the triangle ABC , if $\alpha=40^{\circ}, \beta=60^{\circ}, a=4$. Find $b, c$ ?
Ex: In the triangle ABC , if $\sin \beta=\frac{3}{4}, b=3, a=2$. Find $\sin \alpha$ ?

## The Ambiguous Case:


I) If $x<y$

1) $y \sin \theta<x \Rightarrow 2$ triangles
2) $y \sin \theta=x \Rightarrow 1$ triangle (right)
3) $y \sin \theta>x \Rightarrow$ No triangle
II) If $x \geq y \Rightarrow 1$ triangle

EX: How many triangles ABC can be constructed

1) with $b=2 \sqrt{2}, c=4, \beta=45^{\circ}$ ?
2) with $a=1, b=\sqrt{3}, \quad \alpha=30^{\circ}$ ?
3) with $a=\sqrt{3}, c=1, \quad \gamma=60^{\circ}$ ?
4) with $a=3, b=2, \alpha=140^{\circ}$ ?

Note: In navigation and surveying, the direction or bearing from a point O to a point P equal to the acute angle $\theta$ between the ray OP and the Vertical
line through O , the North-South line


1- A point $\mathbf{P}$ on the level ground is 3 kilometers due north of a point $\mathbf{Q}$. A runner proceeds in the direction $N 25^{\circ} E$ from $\mathbf{Q}$ to a point $\mathbf{R}$, then from $\mathbf{R}$ to $\mathbf{P}$ in the direction $S 70^{\circ} \mathrm{W}$. Find the distance run.

2- Consult the figure. To find the length of the span of a proposed ski lift from A to B to, a surveyor measures the angle DAB to be $25^{\circ}$ and then walks off a distance of 1000 feet to C and measures the angle ACB to be $15^{\circ}$. What is the distance from A to $B$ ?


3- The angle of elevation of an airplane observed by two observers from two points $A$ and $B$ on level ground are $40^{\circ}$ and $35^{\circ}$ respectively. Point A and B are 1000 ft apart and the airplane is between the points, in the same vertical plane. a- How high is the airplane? b- Find the distance between the airplane and the observer at point A .

4- The angle of depression from a balloon to two points A and B on level ground are $52^{\circ}$ and $28^{\circ}$ respectively. Points A and B are 14 miles apart and the balloon is between the points, in the same vertical plane. Find the distance in miles between the balloon and the point A .

