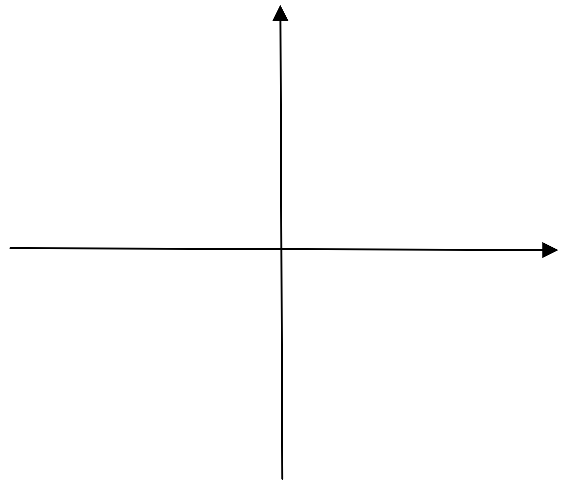
<u>8.3:</u> <u>The Complex Plane ; Demoivre's Theorem</u>

<u>Geometric representation</u> of $x+iy$

<u>Complex plane</u> instead of $xy-$plane



<u>Ex:</u> Plot $2-3i$, $-5+2i$

<u>Note:</u>

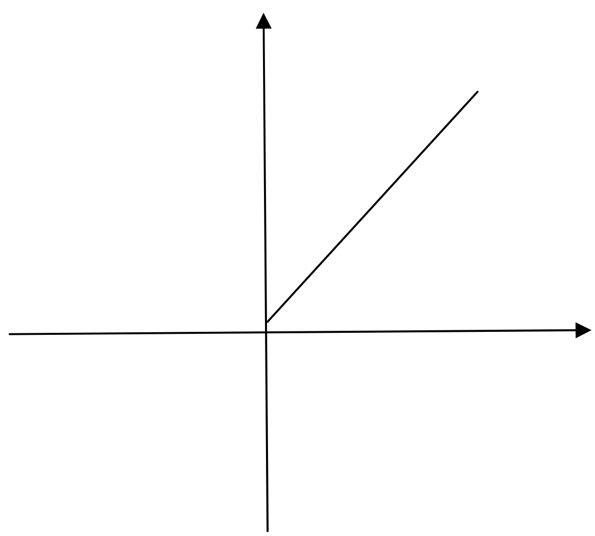
1) $i=\sqrt{-1}$, $i^2=-1$

2) Let $z=x+iy$, then the magnitude $|z|=\sqrt{x^2+y^2}$ [**<u>i.e.</u>** the distance from the origin to the point (x,y)]

<u>Ex:</u> Find the magnitude of $2-3i$

<u>Polar form of a Complex number</u>

Let $z=x+iy$



$x=$

$y=$

$r=$

$\tan\theta=$

$\therefore\ z=$

Ex: Write in polar form

1) $1-i$, 2) $-1+\sqrt{3}i$, 3) $-6i$, 4) $-\sqrt{3}-i$

Theorem: Let

$$z = r_1(\cos\theta_1 + i\sin\theta_1)$$
$$w = r_2(\cos\theta_2 + i\sin\theta_2)$$

Be two complex numbers , then

$zw =$

$\frac{z}{w} =$, $w \neq 0$

Ex: If $z = \sqrt{3}-i$, $w = -\sqrt{3}-i$. Find zw and $\frac{w}{z}$

Ex: If $z = -i$, $w = -1+i$. Find zw and $\frac{z}{w}$

Demoivre's Theorem:

If $z = r(\cos\theta + i\sin)$ is a complex number, then

$z^n =$

Where $n \geq 1$ is a positive integer.

Ex: Find

1) $(-1+\sqrt{3}i)^8$, 2) $(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)^{25}$, 3) $(-2-2i)^{10}$

Complex Roots :

Let $z = r(\cos\theta + i\sin)$ be a complex number. If $z \neq 0$, there are n distinct complex roots of z , given by the formula

$$z_k = \sqrt[n]{r}\,[\cos(\frac{\theta + 2\pi k}{n}) + i\sin(\frac{\theta + 2\pi k}{n})]$$

where $k = 0,1,2,\ldots,n-1$

[**i.e.** finding $z^{\frac{1}{n}}$, $n \geq 1$ positive]

Ex: Find all the complex roots.

1) The complex cube roots of i
2) The complex fourth roots of $\sqrt{3} + i$
3) If $z = 4\sqrt{3} - 4i$, Find a) $z^{\frac{1}{2}}$, b) $z^{\frac{1}{3}}$

Ex: If $z = \sqrt{2}(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ and $w = \cos\frac{\pi}{10} + i\sin\frac{\pi}{10}$, find

a) $z^2 w^5$ b) $\frac{z^2}{w^5}$ (write the answer in ordered pair (r,θ))

Ex: If $z = 3(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$ and $w = 6(\cos(\frac{11\pi}{12}) + i\sin(\frac{11\pi}{12}))$, find

$\frac{w}{z}$ (write the answer in ordered pair (x,y))

Ex: If $z = -16$, find the value of $z^{\frac{1}{4}}$ when $k = 2$
(write the answer in ordered pair (x,y))

Ex: If $z = \sqrt{2}[\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})]$ then find z^8

Ex: Given;

1) $z = \cos(-\pi - \tan^{-1}(\frac{8}{15})) + i\sin(-\pi - \tan^{-1}(\frac{8}{15}))$

2) $z = 8\,[\cos(\frac{3\pi}{2} - \tan^{-1}(-\frac{3}{\sqrt{7}})) + i\,\sin(\frac{3\pi}{2} - \tan^{-1}(-\frac{3}{\sqrt{7}}))]$

3) $z = 15\,[\cos(\tan^{-1}(-\frac{4}{3}) - 8\pi) + i\,\sin(\tan^{-1}(-\frac{4}{3}) - 8\pi)]$

Find $z = x + iy$, write the answer as ordered pair (x, y)