

## 5.2. DERIVATIVES OF LOGARITHMS AND EXPONENTIAL FUNCTIONS

### Algebra Review

(1) Definition of the logarithm:  $\log_a x = y$  iff

(2) Change of Base Formula:  $\log_a u =$

(3) Exponential Formula:  $u^w =$

### Calculus Formulas

(1)  $\frac{d}{dx}(e^x) =$

(2)  $\frac{d}{dx}(\ln x) =$

**Remark 5.2.1.** *The domain of  $f(x) = \ln x$  is  $x > 0$ , so the domain of  $f'(x)$  is*

(3)  $\frac{d}{dx}(\log_a x) =$

(4)  $\frac{d}{dx}(a^x) =$

## Examples

**Example 5.2.1.** If  $f(x) = 2e^x + 5 \ln x$ , find  $f'(x)$

**Example 5.2.2.** Find the derivative of  $y = e^{3x^2} + \ln(4x^5)$

**Example 5.2.3.** If  $f(x) = x^e + e^x$ , find  $f'(x)$

**Example 5.2.4.** If  $f(x) = e^2 + \ln 2$ , find  $f'(x)$

**Example 5.2.5.** If  $f(x) = 2x^3 - e^7 - 5x^c + 4e^x$ , find  $f'(x)$

**Example 5.2.6.** If  $f(x) = -4 \ln x - \ln 6 + 4x^5$ , then  $f'(x)$  equals

- (1)  $-\frac{4}{x} - \frac{1}{6} + 20x^4$
- (2)  $-\frac{1}{4x} - \frac{1}{6} + 20x^4$
- (3)  $-\frac{x}{4} + 20x^4$
- (4)  $-\frac{4}{x} + 20x^4$
- (5) none of these

**Example 5.2.7.** If  $f(x) = x^{10} \ln x$ , then  $f'(x)$  equals

**Example 5.2.8.** Find the derivative of  $y = e^{2x} \ln(x^3)$

**Example 5.2.9.** Find the derivative of  $f(x) = \frac{e^x}{3x - 5x^2}$

**Example 5.2.10.** Find the equation of the tangent line to the graph of  $y = 4 \ln x$  when  $x = 3$ .

**Example 5.2.11.** Select ALL the correct choices regarding the absolute maximum of  $f(x) = 3x - 2x \ln x$  on  $(0, \infty)$ .

- (1) max at  $x = e^{\frac{1}{2}}$
- (2) max of  $2e^{\frac{1}{2}}$
- (3) max at  $x = 2e^{\frac{1}{2}}$
- (4) max of  $e^{\frac{1}{2}}$
- (5) there is no absolute max

**Example 5.2.12.** Select ALL the correct choices for  $f(x) = (x - 7)e^x$ .

- (1)  $f(x)$  is increasing on  $(5, \infty)$
- (2)  $f(x)$  is decreasing on  $(6, \infty)$
- (3) the graph of  $f(x)$  is concave up on  $(6, \infty)$
- (4) the graph of  $f(x)$  has an inflection point at  $x = 5$
- (5)  $f(x)$  has an absolute max at  $x = 6$
- (6)  $f(x)$  has an absolute min at  $x = 5$

**Example 5.2.13.** The demand for souvenir T-shirts sold by a vendor at the Civic Center is given approximately by  $p = 15 - 4 \ln x$ ,  $12 \leq x \leq 40$ , where  $x$  is the number of T-shirts (in thousands) that can be sold at a concert at a price  $\$p$ . If the wholesale cost of the T-shirts is  $\$5$  each, find the profit function,  $w(x)$  (in thousands of dollars).

- (1)  $w(x) = 15x - 4x \ln x - 5$
- (2)  $w(x) = 10x - 4 \ln x^2$
- (3)  $w(x) = 10x - 4x \ln x$
- (4)  $w(x) = 15x - 4x \ln x$
- (5) none of these

**Example 5.2.14.** Find the maximum profit in the previous problem.

Homework: 5.2 p. 330 # 3, 7, 13, 19, 29, 33, 35, 43, 63 work e-grade practice at least 2 times.