8.6. Double Integrals over Rectangular Region

Example 8.6.1. *Evaluate* $\int 12x^3y^4\, dx$ *and* $\int 12x^3y^4\, dy$.

Example 8.6.2. *Evaluate* $\int_{-1}^{1} 12x^3y^4\, dx$ *and* $\int_{-1}^{1} 12x^3y^4\, dy$.

Example 8.6.3. *Evaluate* $\int 3x + 4x^2y - 5y\,dx$ *and* $\int 3x + 4x^2y - 5y\,dy.$

Example 8.6.4. *Evaluate* $\int_{-1}^{2} 3x + 4x^2y - 5y\,dx$ *and* $\int_{-1}^{2} 3x + 4x^2y - 5y\,dy.$
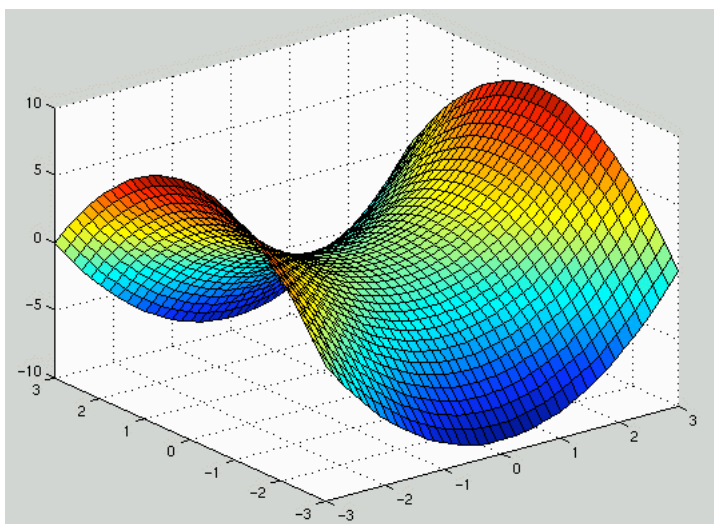
Double Integrals

Example 8.6.5. *Evaluate* $\displaystyle\int_{-1}^{2}\int_{0}^{3} 2x^3y - 6y^2 \, dx \, dy$ *and* $\displaystyle\int_{-1}^{2}\int_{0}^{3} 2x^3y - 6y^2 \, dy \, dx$.

Example 8.6.6. *Evaluate* $\displaystyle\int_1^2 \int_0^1 12y^3 e^x \, dx \, dy.$

Example 8.6.7. *Evaluate* $\displaystyle\iint_R 2x^3 y \, dA$ *where* $R = \{(x,y)|1 \le x \le 2, 0 \le y \le 4\}.$
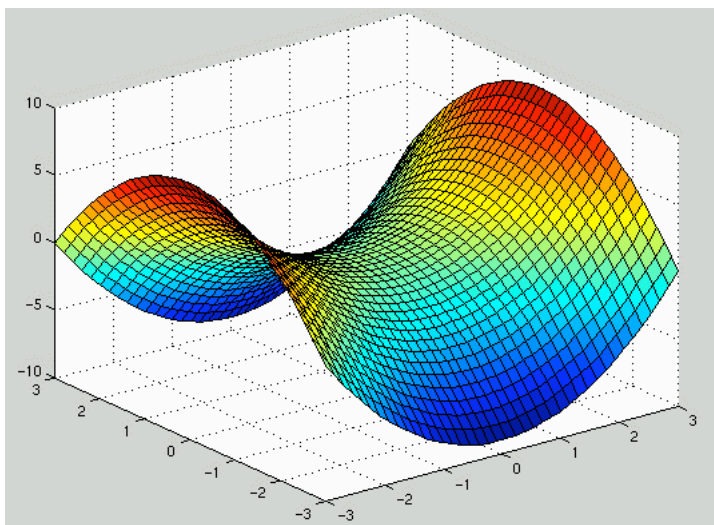
Average Value

Definition 8.6.1. *The* _____ *of the function* $f(x,y)$
over the rectangle $R = \{(x,y)|a \leq x \leq b, c \leq y \leq d\}$ *is*

Example 8.6.8. *Find the average value of* $f(x,y) = 2y^2 - x$ *over* $R = \{(x,y)|-3 \leq x \leq 3, -1 \leq y \leq 1\}$.

Volume

Suppose $z = f(x, y)$ is a function with $z \geq 0$ on the rectangle $R = \{(x, y) | a \leq x \leq b, c \leq y \leq d\}$. Then the volume of the region below the surface $z = f(x, y)$, above the xy-plane in the rectangle R is found by

Example 8.6.9. *Find the volume of the solid under the graph of $f(x, y) = 2 - x^2 + 2y^2$ and over the rectnale $R = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$.*

Homework: 8.6 p.547 # 3, 5, 11, 15, 17, 21, 25, 27 work e-grade practice at least 2 times.