# 4.2. First Derivative and Graphs

## DEFINITIONS

$y = f(x)$ is a function with domain $D$.

(1) (Intuitive Idea) A function is increasing on the interval $(a, b)$ if as you trace it left to right the graph rises. It is decreasing if the graph falls from left to right.

Using info from calculus,

(a) If $f'(x) > 0$ on the interval then the function is ________________________.

(b) If $f'(x) < 0$ on the interval then the function is ________________________.

(2) $f$ has an ______________________________ or ______________________________ *at* $x = c$ if $f(c) \geq f(x)$ for $x$ "close enough" to $c$. $f(c)$ is the

______________________________.

(3) $f$ has an ______________________________ or ______________________________ *at* $x = c$ if $f(c) \leq f(x)$ for $x$ "close enough" to $c$. $f(c)$ is the

______________________________.

(4) "close enough" to $c$ means there is an open interval around $c$ where the statement is true. This open interval can be very small.

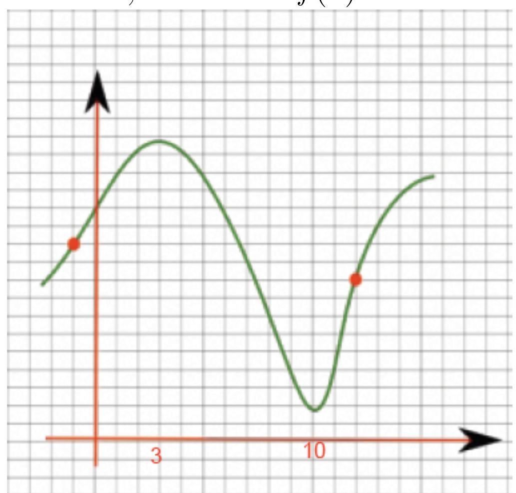
(5) The local minimum and local maximum values are called the

______________________________ of $f$ and the points where local

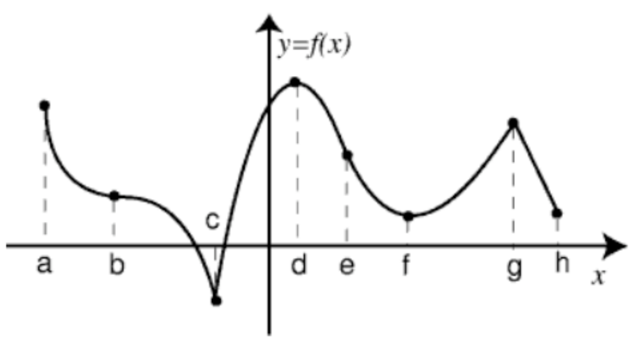
extrema occur are called ______________________________.

# Example

**Example 4.2.1.** *Find the intervals where the functions is increasing, where decreasing, where $f'(x) > 0$, where $f'(x) < 0$, where $f'(x) = 0$, where $f'(x)$ does not exist, where $f(x)$ has a local minimum, and where $f(x)$ has a local maximum.*



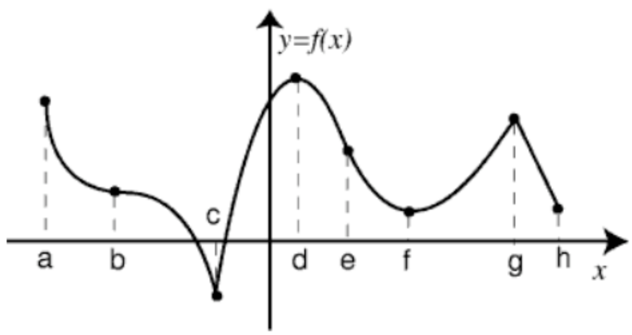

**Example 4.2.2.** *Find the intervals where the functions is increasing, where decreasing, where $f'(x) > 0$, where $f'(x) < 0$, where $f'(x) = 0$, where $f'(x)$ does not exist, where $f(x)$ has a local minimum, and where $f(x)$ has a local maximum.*



## The First Derivative Test

**Theorem 4.2.1** (Fermat's Theorem)**.** *If $f$ has a local extreme at $c$, then $f'(c) = 0$ or $f'(c)$ is undefined.*

*This tells us that the only possible places where $f$ may have a local extreme is where the derivative is equal to 0 or is undefined. These values are called the* **critical number(s)** *of a function. Note that a critical number does not have to be a local extreme, but a local extreme has to be a critical number.*




**First Derivative Test:**

Find all critical numbers of $f$. Keep in mind that all critical numbers must be in the domain of $f$.

(1) If $f'$ is positive to the left of $c$ and negative to the right of $c$, then $f$ has a local maximum at $c$.
(2) If $f'$ is negative to the left of $c$ and positive to the right of $c$, then $f$ has a local minimum at $c$.
(3) If $f'$ does not change signs at $c$, then $f$ does not have a local extreme at $c$.

## Using the First Derivative Test

(1) Find the domain of $f$.
(2) Find all critical numbers of $f$.
(3) Place all critical numbers AND values where $f$ is undefined on a number line. These numbers will separate the number line into intervals.
(4) Determine the sign of $f'$ on each interval on the number line.
(5) Use the information in 4 to determine intervals where $f$ is increasing, decreasing, and where local extremes occur.

**Example 4.2.3.** *Where is $f(x) = -2x^3 + 3x^2 + 120x$ increasing? decreasing?*

**Example 4.2.4.** *Find the local extrema of $f(x) = -2x^3 + 3x^2 + 120x$.*

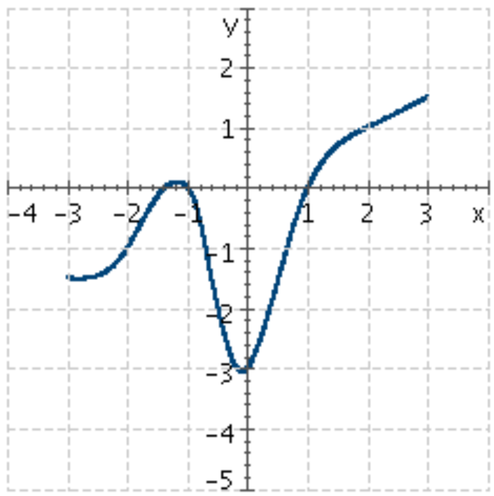
**Example 4.2.5.** *Find the local extrema of $f(x) = -5x^2 + 10$.*

*(1) a local minimum at $x = 0$*
*(2) a local maximum at $x = 10$*
*(3) a local minimum at $x = 10$*
*(4) a local maximum at $x = 0$*

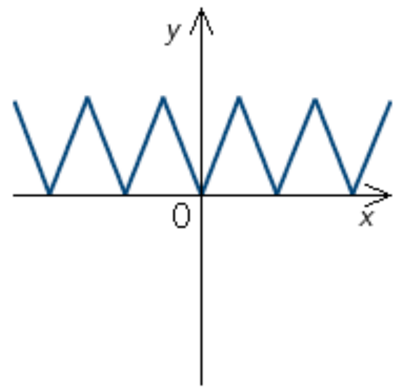
# Graphs v.s. Derivatives

**Example 4.2.6.** *Sketch a possible graph for the derivative of $f$ on the given the graph of $y = f(x)$.*

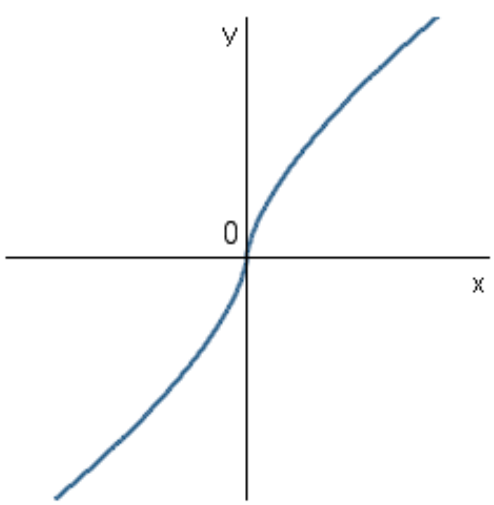
*(a)*



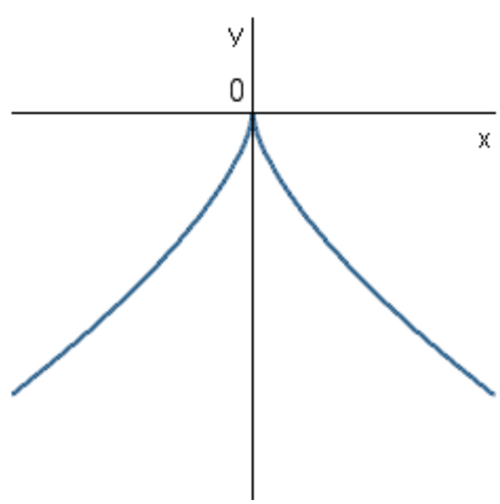

*(b)*



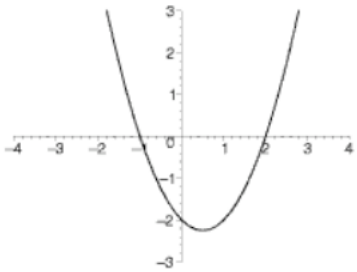

*(c)*

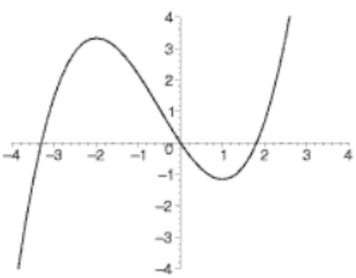



*(d)*

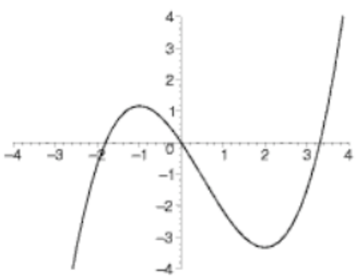


**Example 4.2.7.** *The graph of the derivative, $f'(x)$, is given below. Select a possible graph of $f(x)$.*

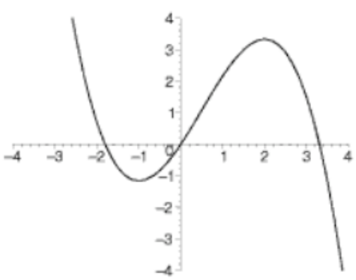




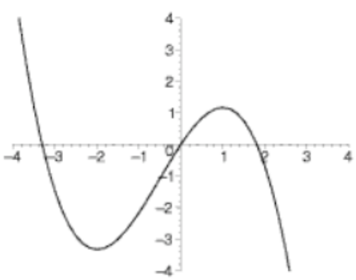
*(a)*



*(b)*



*(c)*



*(d)*

Homework: 4.2 p. 254 # 1-8, 13, 19, 33, 45-50, 51, 61 work e-grade practice at least 2 times.