5.1. The Constant e and Continuous Compound Interest

Definition 5.1.1. *An* ________________________________ *is a function of the form* $f(x) = a^x$ *where* a *is a real number with* $a > 0$ *and* $a \neq 0$*.*

Definition 5.1.2. *The number e is defined by*

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

equivalently

$$e = \lim_{s\to 0}(1+s)^{1/s}$$

e *is an* **irrational number** *and is approximately* 2.718

Logarithmic Functions

Definition 5.1.3. *The logarithm of* x *with respect to the base* a *is defined by*

$y = \log_a x$ *if and only if* ______________________________

Properties of Logarithmic Functions

(1) Special logarithms you should quickly recognize and/or evaluate:

(a) $\log_a 1 =$

(b) $\log_a a =$

(c) $\log x =$

(d) $\ln x =$

(2) Since $f(x) = a^x$ and $g(x) = \log_a x$ are one-to-one:

(a) $a^u = a^v$ if and only if ______________

(b) $\log_a u = \log_a v$ if and only if ______________

(3) Since $f(x) = a^x$ and $g(x) = \log_a x$ are inverses of each other:

(a) $\log_a a^u =$

(b) $a^{\log_a u} =$

Example 5.1.1. *Evaluate* $e^{\ln 4 + \ln 3}$

(4) Operations:

(a) $\log_a(mn) =$

(b) $\log_a(m/n) =$

(c) $\log_a(m^n) =$

(5) Change of base formula: If $b > 0$ and $b \neq 0$, then $\log_a x =$

In particular, $\log_a x =$

(6) $f(x) = \log_a x$ is only defined for

(7) The graphs of exponents and logarithms.

Example 5.1.2. *For the following $x > 0$, $x \neq 1$, $y > 0$ and $y \neq 1$. True or false?*

(a) $\log_x 1 = 0$

(b) $\log xy = \log x \cdot \log y$

(c) $\log_y x = \dfrac{\log y}{\log x}$

(d) $\log \dfrac{x}{y} = \dfrac{\log x}{\log y}$

(e) $\log_5 5^{-3} = -3$

(f) $\log_{-5}(-5)^3 = 3$

(g) $\ln x \to \infty$ *as* $x \to \infty$

(h) $\ln(-a)$ *is defined*

(i) $\ln e^x = 1$

Interest Compounded n times per year

The __________________________________, A, is amount in account at the end of given time period of an account.

The __________________________________ or ________________________, P, is the amount initially deposited.

The ____ or __________________________________, r, is the rate for the full year in decimal form.

n is the number of times per year the account is ________________________, i.e. the number of times per year the interest is calculated and added to the account.

t is the number of years the account is held.

FORMULA for A:

Example 5.1.3. *Find the amount that results from* \$350 *invested at* 12% *compounded quarterly after a period of* 9 *years.*

(1) $350\left(1+\dfrac{0.12}{4}\right)^{36}$

(2) $\dfrac{350}{\left(1+\frac{0.12}{4}\right)^{36}}$

(3) $350\left(1+\dfrac{0.12}{4}\right)^{9}$

(4) $\dfrac{350}{\left(1+\frac{0.12}{4}\right)^{9}}$

Interest Continuously Compounded

An account that is ________________________________ is the value the previous formula approaches when $n \to \infty$.

FORMULA for A:

Example 5.1.4. *If* $\$4,765$ *is invested at* 9.8% *compounded continuously, what is the amount in 5 years?*

(1) $\frac{4765}{e^{0.49}}$

(2) $4765e^{4.9}$

(3) $4765e^{0.49}$

(4) $\frac{4765}{e^{4.9}}$

(5) none of these

Example 5.1.5. *What continuously compounded interest rate will double an investment in 8 years?*

(1) $\ln \frac{1}{4}$

(2) $\ln 4$

(3) $\frac{\ln 2}{8}$

(4) $\frac{\ln 8}{2}$

(5) none of these

Example 5.1.6. *What interest rate, compounded continuously, will take an investment of* $\$10,000$ *to* $\$40,000$ *in 5 years?*

Example 5.1.7. *How long will it take* $\$85,000$ *to grow to* $\$100,000$ *at* 7% *annual interest compounded continuously?*

Homework: 5.1 p. 320 # 17, 19, 27 work e-grade practice at least 2 times.