6.5. Definite Integral; Fundamental Theorem of Calculus

We assume for this section that f is a continuous function on the interval $[a, b]$ and n is a positive integer.

(1) We may subdivide the interval $[a, b]$ into n subintervals. Denote the endpoints of the subintervals $x_0, x_1, x_2, \ldots, x_n$ where $a = x_0$, $b = x_n$ and $x_{i-1} < x_i$. This is called a ________________________ of the interval $[a, b]$.

(2) It is most common to choose the subintervals to all have the same width.

(3) $\Delta x_i =$ ________________________ $=$

(4) It is most common to choose the subintervals to all have the same width, so

(5) x_i^* denotes a chosen number in the interval $[x_{i-1}, x_i]$.

(6) $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ is a __________________________________

(7) If $x_i^* = x_{i-1}$, then the Riemann sum is called the ____________ Riemann sum.

(8) If $x_i^* = x_i$, then the Riemann sum is called the _______________ Riemann sum.

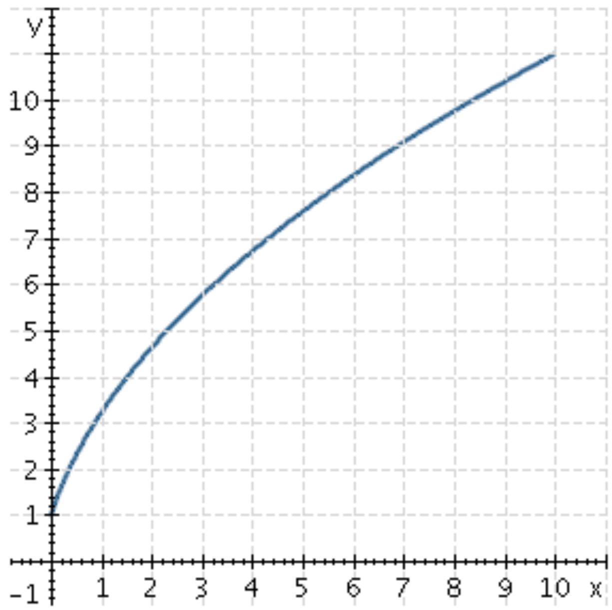
(9) If $x_i^* = \dfrac{x_i - x_{i-1}}{2}$, then the Riemann sum is called the _________________ Riemann sum.

(10) If x_i^* is where the maximum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the _______________ Riemann sum.

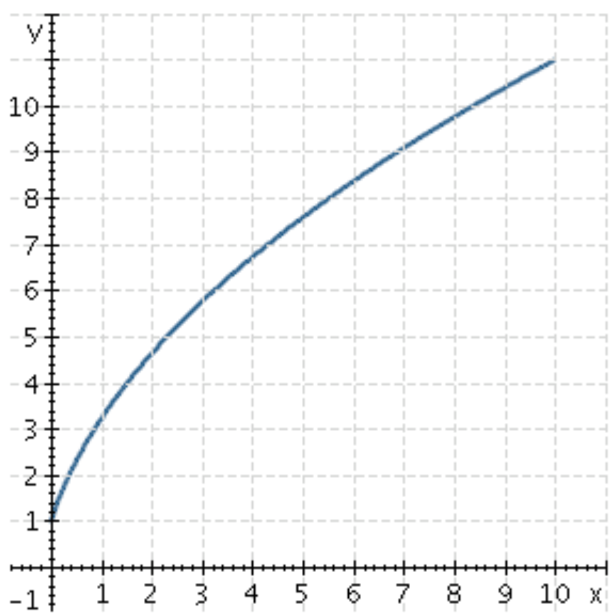
(11) If x_i^* is where the minimum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the _______________ Riemann sum.

Graphs

Example 6.5.1. *Left rule on* $[0, 10]$ *with 5 equal subintervals.*



Example 6.5.2. *Midpoint rule on* $[0, 10]$ *with 5 equal subintervals.*



Example 6.5.3. *Use the rectangle (mid-point) rule, with $n = 2$ to approximate* $\int_{-4}^{2}(x - 2x^2)\,dx$

Integrals

(1) The **definite integral of f from a to b** is defined by

Note that when the subintervals are chosen so the width of each is $\dfrac{b-a}{n}$, then this is equivalent to

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

(a) $\int$ is called the ______________________

(b) a and b are the ______________________________ with b being the ______________________ while a is the ______________________

(c) $f(x)$ is the ______________________

(d) the process of finding the integral is ______________________

Properties

(1) If ____ then $\int_a^b f(x)\,dx$ is the exact area between the curve and the x axis over the interval $[a, b]$.

(2) If ____ then $\int_a^b f(x)\,dx$ is -1 times the exact area between the curve and the x axis over the interval $[a, b]$.

(3) $\displaystyle\int_a^b f(x)\,dx =$

(4) $\displaystyle\int_a^a f(x)\,dx =$

(5) $\displaystyle\int_a^b c\,dx =$

(6) $\displaystyle\int_a^b [f(x) + g(x)]\,dx =$

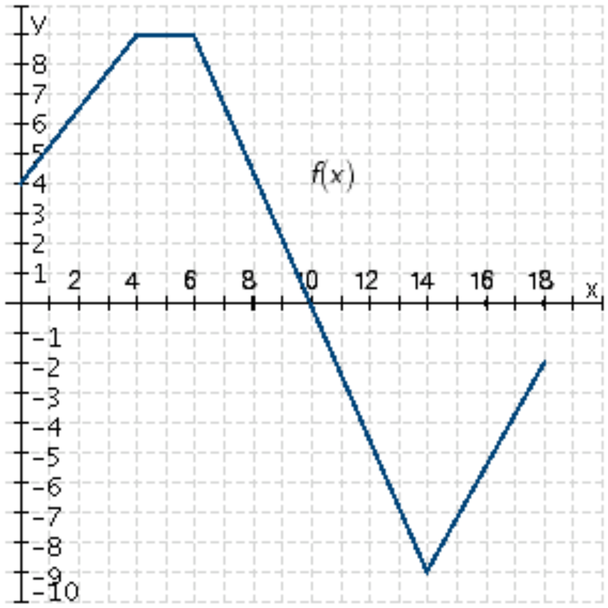
(7) $\displaystyle\int_a^b cf(x)\,dx =$

(8) $\displaystyle\int_a^b [f(x) - g(x)]\,dx =$

(9) $\displaystyle\int_a^c f(x)\,dx =$

Examples

Example 6.5.4. *Use the graph to find the following integrals.*



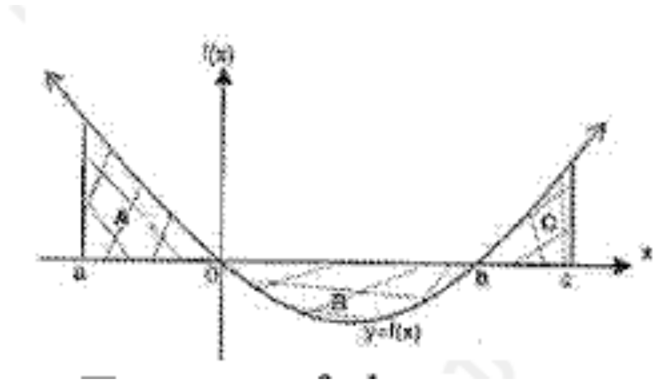
(1) $\int_{0}^{4} f(x)\,dx$

(2) $\int_{14}^{16} f(x)\,dx$

(3) $\int_{6}^{14} f(x)\,dx$

(4) $\int_{6}^{4} f(x)\,dx$

Example 6.5.5. *Select ALL correct choices for the following graph with Area $A = 6$, Area $B = 15$, Area $C = 3$.*



(1) *none of these*
(2) $\int_b^c f(x)\,dx = 3$
(3) $\int_a^c f(x)\,dx = -6$
(4) $\int_b^0 \frac{f(x)\,dx}{5} = -3$
(5) $\int_b^a f(x)\,dx = 9$

Example 6.5.6. *Given $\int_3^5 f(x)\,dx = 5$ and $\int_1^5 f(x)\,dx = 7$ find $\int_1^3 f(x)\,dx$.*

The Fundamental Theorem of Calculus

Theorem 6.5.1 (FTC)**.** *Assume f is continuous on $[a, b]$.*

(1) Then the function g defined by

is differentiable on (a, b) and $g'(x) = f(x)$.

(2) If $F(x)$ is an antiderivative of f then

Definition 6.5.1. *The most general antiderivative of $f(x)$ is also called the*

____ *and is denoted*

Thus if $F(x)$ is an antiderivative of f, then

Examples

Example 6.5.7. *Evaluate* $\int_{-3}^{4} (3x^2 - 4x)\,dx$

Example 6.5.8. *Integrate* $\int_{0}^{4} 3 + \sqrt{x}\,dx$

Example 6.5.9. *Integrate* $\int_{-2}^{4} e^{-5x}\,dx$

Example 6.5.10. *Evaluate* $\displaystyle\int_{-3}^{0} \frac{x}{16 - x^2}\,dx$

Example 6.5.11. *Evaluate* $\displaystyle\int_{3}^{0} x\sqrt{x^2 + 16}\,dx$

Example 6.5.12. *An oil well starts out producing oil at a rate of 60,000 barrels per year, and the production rate decreases by 4,000 barrels per year. Thus, if $P(t)$ is the total production (in thousands of barrels) in t years, the rate of change of production is $P'(t) = 60 - 4t$, $0 \leq t \leq 15$. Find the total production of oil (in thousands of barrels) over the first 7 years of operation.*

(1) 32
(2) 42
(3) 322
(4) 420

Average Function Value

The Average Value of f over the interval $[a, b]$ is defined as

Examples

Example 6.5.13. *Find the average value of* $g(t) = -6t^2 + 4t$ *over the interval* $[-2, 3]$.

Example 6.5.14. *Suppose the inventory,* I*, of a certain item,* t *months after the first of the year, is* $I(t) = 3 + 18t - 3t^2$, $0 \leq t \leq 12$*. What is the average inventory for the first 6 months of the year?*

Homework: 6.5 p. 430 # 1, 3, 5, 13, 17, 21, 23, 33, 39, 47a, 51a, 53 first part, 79, 83, 85, 103, work e-grade practice at least 2 times.