

## 7.2. APPLICATIONS IN BUSINESS AND ECONOMICS

**Definition 7.2.1.** Suppose  $x$  represents a possible numerical outcome of an experiment and that  $[c, d]$  represents an interval of possible outcomes of the experiment. A 

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function,  $y = f(x)$ , is a function that helps us determine the probability that an outcome of  $x$  will occur in the interval  $[c, d]$

This functions must satisfy the following properties:

(1)

(2)

(3) When  $[c, d]$  is an interval of real numbers then the probability of outcome  $x$  during the interval  $[c, d]$  is

**Example 7.2.1.** The shelf life (in years) of a certain brand of clock radio is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{(x+3)^2} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that a randomly selected clock radio has a shelf life from 2 to 6 years?

**Example 7.2.2.** *The shelf life (in years) of a certain brand of clock radio is a continuous random variable with probability density function*

$$f(x) = \begin{cases} \frac{4}{(x+4)^2} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

*What is the probability that a randomly selected clock radio has a shelf life of 1 year or less?*

**Example 7.2.3.** *A manufacturer guarantees a product for 5 years. The time to failure of that product after it is sold is given by the probability density function*

$$f(t) = \begin{cases} 0.03e^{-0.03t} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

*where  $t$  is time in years. What is the probability that a buyer chosen at random will have a product failure during the warranty period?*

**Example 7.2.4.** *The time to failure of a product after it is sold is given by the probability density function*

$$f(t) = \begin{cases} 0.15e^{-0.15t} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where  $t$  is time in months. What is the probability that a buyer chosen at random will have a product failure during the second year after purchase?

## Income Stream

**Definition 7.2.2.** *In many applications it is assumed that income is received in a \_\_\_\_\_ rather than in discrete points of time. The rate of change of the continuous income stream is called the \_\_\_\_\_.*

**Theorem 7.2.1.** *If  $f(t)$  is the rate of flow of a continuous income stream, then the total income produced from the time  $t = a$  to  $t = b$  is*

**Example 7.2.5.** *Find the total income (in thousands of dollars) produced by a vending machine in its first 3 years of operation if the rate of the flow of its continuous income stream is given by  $f(t) = 6e^{0.05t}$ , where  $t$  is the time in years since the installation of the machine and  $f(t)$  is in thousands of dollars.*

- (1)  $120(e^{0.15} - e^{0.05})$
- (2)  $120e^{0.15} - 1$
- (3)  $120e^{0.15}$
- (4)  $120(e^{0.15} - 1)$
- (5) none of these

**Definition 7.2.3.** Suppose an income with a rate of flow  $f(t)$  is invested in a continuously compounded account with an interest rate  $r$ . The \_\_\_\_\_,  $FV$ , at the end of  $T$  years later is given by

### Consumers' and Producers' Surplus

**Definition 7.2.4.** Suppose  $p = D(x)$  is a price-demand equation for a product. Then the \_\_\_\_\_,  $CS$ , at a price level of  $\bar{p}$

where  $\bar{x}$  is the demand at price  $\bar{p}$ .

**Definition 7.2.5.** Suppose  $p = S(x)$  is a supply-supply equation for a product. Then the \_\_\_\_\_,  $PS$ , at a price level of  $\bar{p}$

where  $\bar{x}$  is the demand at price  $\bar{p}$ .

**Definition 7.2.6.** The point(s) where the price-demand equation and the price-supply equation intersect is called the \_\_\_\_\_ and

\_\_\_\_\_

**Example 7.2.6.** *The price-demand equation for a produce is  $p = D(x) = 13 - \frac{2}{3}x$ . If the equilibrium price is \$7, what is the consumer's surplus in dollars?*

**Example 7.2.7.** *The price-demand equation for a produce is  $p = D(x) = \frac{21}{3x + 4}$ . If the equilibrium quantity is 1 unit, what is the consumer surplus in dollars?*

**Example 7.2.8.** *The price-supply equation for a produce is  $p = S(x) = 2x^2 + x + 5$ . If the equilibrium price is \$15, what is the producer's surplus in dollars?*

**Example 7.2.9.** *For a certain product, the price-demand and price-supply equations are  $p = D(x) = 20 - 3x^2$  and  $p = S(x) = 3x + 2$  respectively. Find the consumer surplus at the equilibrium price level.*

- (1) 6
- (2) 12
- (3) 16
- (4) 48
- (5) none of these

Homework: 7.2 p. 467 # 7, 11, 15, 19, 37, 41, 47 work e-grade practice at least 2 times.