## 8.3. Maxima Minima

**Theorem 8.3.1.** *Let $z = f(x, y)$ be a function of two variables.*

*GIVEN:*

(1) $f_x(a, b) = 0$ *and* $f_y(a, b) = 0$ *($(a, b)$ is a* ______________________ *for $f$)*

(2) *All second partial derivative exist around the point $(a, b)$.*

(3) $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $C = f_{yy}(a, b)$

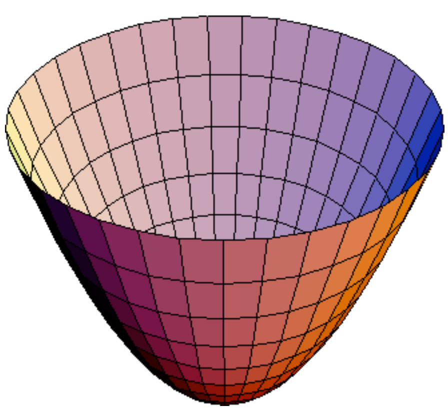
*THEN:*

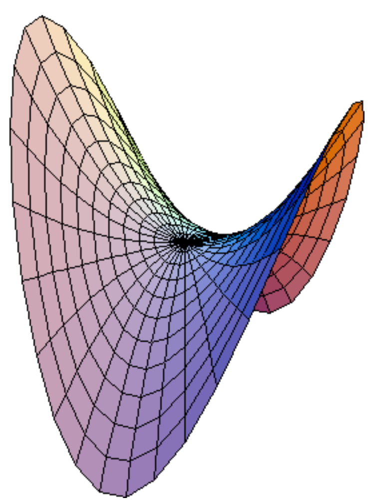
*case 1: If* $AC - B^2 > 0$ *and* $A < 0$*, then* $f(a, b)$ *is a* ______________________________

*case 2: If* $AC - B^2 > 0$ *and* $A > 0$*, then* $f(a, b)$ *is a* ______________________________

*case 3: If* $AC - B^2 < 0$*, then $f$ has a* ______________________________ *at $(a, b)$.*

*case 4: If* $AC - B^2 = 0$ *the test fails –* ______________________________





# Examples

**Example 8.3.1.** *Find all local extrema and saddle points of* $f(x, y) = 2x^2 - 2xy + y^2 - 4x + 6y - 3$

**Example 8.3.2.** *Find all local extrema and saddle points of* $f(x, y) = 8x + 6y - 17$

**Example 8.3.3.** *Find all local extrema and saddle points of* $f(x, y) = -2x^2 + 4xy - 3y^2 - 4x + 2y - 3$

**Example 8.3.4.** *Find all local extrema and saddle points of* $f(x, y) = xy + x - y$

**Example 8.3.5.** *Find all local extrema and saddle points of* $f(x, y) = 3y^2 - 2x^3 - 24x - 3y - 21$

**Example 8.3.6.** *Find all local extrema and saddle points of* $f(x, y) = 2x^3 - 2xy + 2y$

**Example 8.3.7.** *Find all local extrema and saddle points of* $f(x, y) = -2x^2 + 4xy - 3y^2 - 4x + 2y - 3$

**Example 8.3.8.** *The cost function, C (in hundreds of dollars), of producing two products is* $C(x, y) = 2x^2 + 3y^2 - 4xy + 4x - 8y + 20$*, where* $x$ *is the quantity of product A and* $y$ *is the quantity of product B.*

(1) *How many of each product should be produced to minimize cost.*
(2) *Find the minimum cost of producing these products.*

Homework: 8.3 p. 516 # 9, 13, 17, 21, 23, 29, 31 work e-grade practice at least 2 times.