4.1. First Derivative and Graphs

DEFINITIONS

$y = f(x)$ is a function with domain D.

(1) (Intuitive Idea) A function is increasing on the interval (a, b) if as you trace it left to right the graph rises. It is decreasing if the graph falls from left to right.

Using info from calculus,

(a) If $f'(x) > 0$ on the interval then the function is __________________________.

(b) If $f'(x) < 0$ on the interval then the function is __________________________.

(2) f has an ________________________________ or ________________________________ *at* $x = c$ if $f(c) \geq f(x)$ for x "close enough" to c. $f(c)$ is the

________________________________.

(3) f has an ________________________________ or ________________________________ *at* $x = c$ if $f(c) \leq f(x)$ for x "close enough" to c. $f(c)$ is the

________________________________.

(4) "close enough" to c means there is an open interval around c where the statement is true. This open interval can be very small.

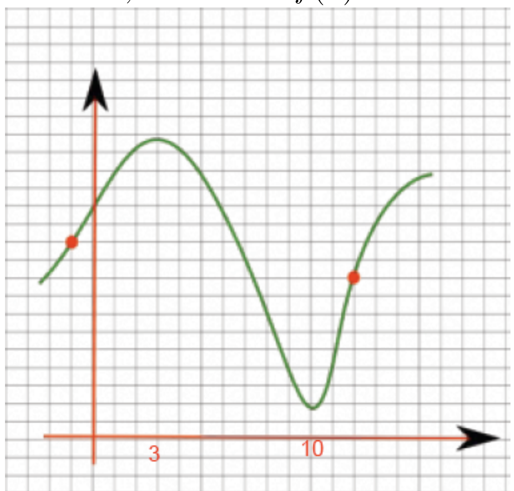
(5) The local minimum and local maximum values are called the

________________________________ of f and the points where local

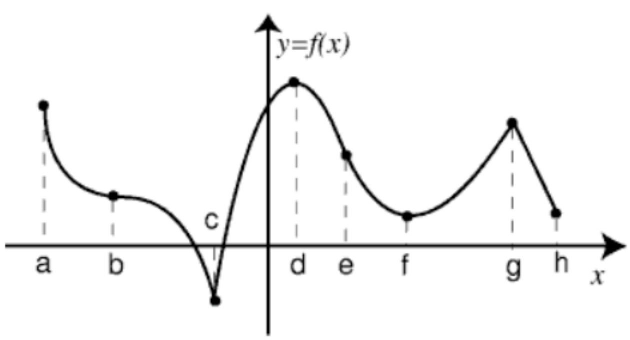
extrema occur are called ________________________________.

Example

Example 4.1.1. *Find the intervals where the functions is increasing, where decreasing, where $f'(x) > 0$, where $f'(x) < 0$, where $f'(x) = 0$, where $f'(x)$ does not exist, where $f(x)$ has a local minimum, and where $f(x)$ has a local maximum.*



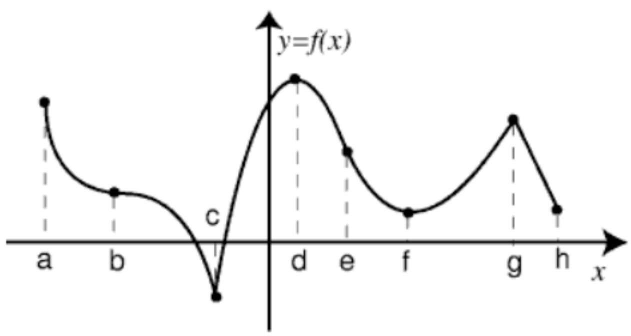
Example 4.1.2. *Find the intervals where the functions is increasing, where decreasing, where $f'(x) > 0$, where $f'(x) < 0$, where $f'(x) = 0$, where $f'(x)$ does not exist, where $f(x)$ has a local minimum, and where $f(x)$ has a local maximum.*



The First Derivative Test

Theorem 4.1.1 (Fermat's Theorem)**.** *If f has a local extrema at c, then $f'(c) = 0$ or $f'(c)$ is undefined.*

This tells us that the only possible places where f may have a local extrema is where the derivative is equal to 0 or is undefined. These values are called the **critical number(s)** *of a function. Note that a critical number does not have to be a local extrema, but a local extrema has to be a critical number.*



<u>First Derivative Test:</u>
Find all critical numbers of f. Keep in mind that all critical numbers must be in the domain of f.

(1) If f' is positive to the left of c and negative to the right of c, then f has a local maximum at c.
(2) If f' is negative to the left of c and positive to the right of c, then f has a local minimum at c.
(3) If f' does not change signs at c, then f does not have a local extreme at c.

Using the First Derivative Test

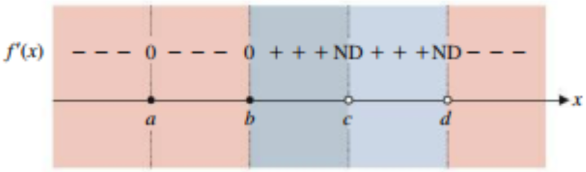
(1) Find the domain of f.
(2) Find all critical numbers of f.
(3) Place all critical numbers AND values where f is undefined on a number line. These numbers will separate the number line into intervals.
(4) Determine the sign of f' on each interval on the number line.
(5) Use the information in 4 to determine intervals where f is increasing, decreasing, and where local extremes occur.

Example 4.1.3. *Where is* $f(x) = -2x^3 + 3x^2 + 120x$ *increasing? decreasing?*

Example 4.1.4. *Find the local extrema of* $f(x) = -2x^3 + 3x^2 + 120x$.

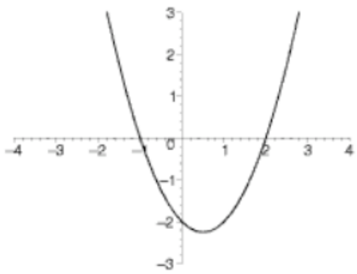
Example 4.1.5. *Find the local extrema of* $f(x) = 2x + \dfrac{128}{x}$.

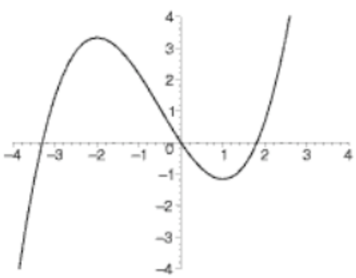
Example 4.1.6. *Assume $f(x)$ is continuous on $(-\infty, \infty)$ and has critical numbers at $x = a,\ b,\ c,$ and d. Use the sign chart for $f'(x)$ to determine whether f has a local maximum, local minimum, or neither at each critical number.*



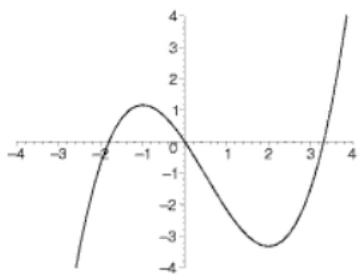
Graphs v.s. Derivatives

Example 4.1.7. *The graph of the derivative, $f'(x)$, is given below. Select a possible graph of $f(x)$.*





(a)



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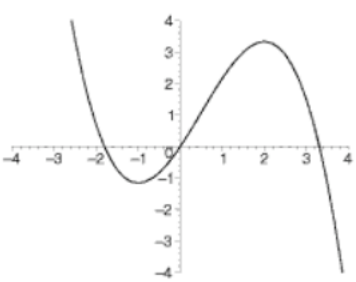

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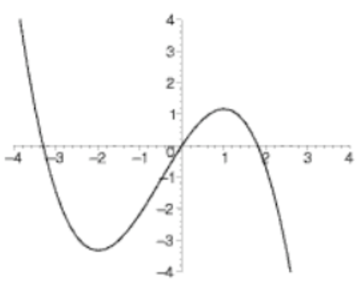

(d)