4.5. Absolute Maxima and Minima

DEFINITIONS

$y = f(x)$ is a function with domain D.

(1) f has an ______________________ or ______________________

at $x = c$ if $f(c) \geq f(x)$ for all x in D. $f(c)$ is the ______________________.

(2) f has an ______________________ or ______________________

at $x = c$ if $f(c) \leq f(x)$ for all x in D. $f(c)$ is the ______________________.

(3) The minimum and maximum values are called the ______________________ of f.

(4) f has an ______________________ or ______________________

at $x = c$ if $f(c) \geq f(x)$ for x "close enough" to c. $f(c)$ is the ______________________
.

(5) f has an ______________________ or ______________________

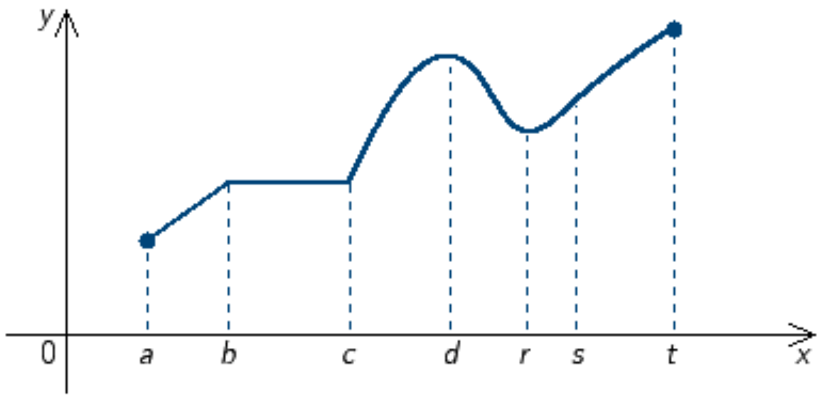
at $x = c$ if $f(c) \leq f(x)$ for x "close enough" to c. $f(c)$ is the ______________________.

(6) "close enough" to c means there is an open interval around c where the statement is true. This open interval can be very small.

(7) The local minimum and local maximum values are called the ______________________ of f.

4.5. Examples

Example 4.5.1. *Find all absolute and local extrema and where they occur.*



Example 4.5.2. *Select ALL the correct choices* $f(x) = 2 - 4x - \dfrac{4}{x}$ *over the interval* $(-\infty, 0)$

(1) $f(x)$ *has no maximum*
(2) $f(x)$ *has no minimum*
(3) $f(x)$ *has an absolute maximum at* $x = -1$
(4) $f(x)$ *has an absolute minimum at* $x = -1$
(5) $f(x)$ *has an absolute maximum of* 2
(6) $f(x)$ *has an absolute minimum of* 2

Useful Theorems

Theorem 4.5.1 (Second Derivative Test)**.** *Suppose $y = f(x)$ is such that $f'(c) = 0$ (and f is twice differentiable around c).*

(1) If $f''(c) > 0$ then ______________________________

(2) If $f''(c) < 0$ then ______________________________

(3) If $f''(c) = 0$ or $f''(x)$ does not exist, then ______________________________

Theorem 4.5.2. *If $f(x)$ has only one critical number in some interval I, then statements (1) and (2) in Theorem 4.5.1 are* <u>*absolute*</u> *extrema.*

Example 4.5.3. *Find the local extrema of the function of $f(x) = x^3 - 4x^2 - 3x - 10$ using the second derivative test.*

Example 4.5.4. *If $f(x)$ is continuous on $(-\infty, \infty)$ such that $f'(7) = 0$ and $f''(7) = -2$, then*

a) $f(x)$ has a local maximum at $x = 7$
b) $f(x)$ has a local minimum at $x = 7$
c) $f(x)$ has no local extrema at $x = 7$
d) we are unable to determine if there is a local extrema at $x = 7$

Example 4.5.5. *If $f(x)$ is continuous on $(-\infty, \infty)$ such that $f'(7) = 1$ and $f''(7) < 0$, then*

a) $f(x)$ has a local maximum at $x = 7$
b) $f(x)$ has a local minimum at $x = 7$
c) $f(x)$ has no local extrema at $x = 7$
d) we are unable to determine if there is a local extrema at $x = 7$

Example 4.5.6. *Find the absolute extrema of* $f(x) = 5 \ln x - x$ *over* $(0, \infty)$

Example 4.5.7. *Find the the absolute extrema of* $f(x) = 4x^4 - 5$.

Theorem 4.5.3 (The Extreme Value Theorem). *If f is continuous on the closed interval $[a, b]$, then f will attain a minimum and a maximum in the interval.*

In other words, if you consider the interval $[a, b]$ as the domain of f, there will be at least one number c in $[a, b]$ where $f(c)$ is the maximum, and at least one number d in $[a, b]$ where $f(d)$ is the minimum.

Closed Interval Method

To find the *absolute* minimum and maximum values of a *continuous* function f on a *closed interval* $[a, b]$:

Step 1. Find the critical numbers of f in (a, b).

Step 2. Find the function value at all critical value(s) found in step 1.

Step 3. Find $f(a)$ and $f(b)$.

Step 4. The largest value from steps 2 and 3 is the maximum value and the smallest value from steps 2 and 3 is the minimum value.

Examples

Example 4.5.8. *Find all critical values and absolute extrema on the given interval.*

$f(x) = 6x - x^2$, $[-1, 4]$

(1) min value is -7*, max value is* 9
(2) min value is -7*, max value is* 40
(3) min value is -5*, max value is* 8
(4) min value is -5*, max value is* 40

Example 4.5.9. *Find all critical values and absolute extrema on the given interval.*

$$f(x) = \frac{x^2 - 4}{x^2 + 4},\ [-4, 4]$$

Example 4.5.10. *Find all the absolute extrema of* $f(x) = \ln x$ *over the interval* $[1, 2]$.