

6.1. AREA BETWEEN CURVES

Theorem 6.1.1 (Area between two curves). *If $f(x) \geq g(x)$ for all x in $[a, b]$, then the area between the graphs of f and g over the interval $[a, b]$ is*

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n [f(x_i^*) - g(x_i^*)] \Delta x = \int_a^b f(x) - g(x) dx$$

In general, the area between the graphs of f and g over the interval $[a, b]$ is

Steps for Finding Area Between Two Curves

- (1) Set both functions equal to each other and find intersection points.
- (2) If interval was given, check if any intersection point(s) you find in step (1) falls inside of the given interval. If no interval was given, then plot all intersection points on a number line (in this case, you should have at least two intersection points).
- (3) Test a value from each subinterval you have into both functions to determine which function is greater within that subinterval.
- (4) Set up an integral for each subinterval and take the larger function minus the smaller function over that subinterval. Add all your integrals together.

Examples

Example 6.1.1. *Find the area between the graphs $y = -6x - 2$ and $y = 10$ for $-2 \leq x \leq 2$.*

Example 6.1.2. Find the area between the graphs $y = -6x - 2$ and $y = 10$ for $-4 \leq x \leq 0$.

Example 6.1.3. Find the area between the graphs $y = -4x^2 - 8x + 5$ and $y = 3x^2 - 8x - 2$.

Example 6.1.4. $y = 1 + \sqrt{x}$ and $y = \frac{3+x}{3}$